Thermal Buckling Analysis of Laminated Composite Shell Panel Embedded with Shape Memory Alloy Fibre under TD and TID

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by

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ROURKELA

CERTIFICATE

This is to certify that the work in this thesis entitled "**Thermal Buckling Analysis of Laminated Composite Shell Panel Embedded with Shape Memory Alloy (SMA) Fibre under TD and TID**" by **Mr. Rohit Kumar Singh** (212ME1284) has been carried out under my supervision in partial fulfilment of the requirements for the degree of **Master of Technology** in Mechanical Engineering with **Machine Design and Analysis** specialization during session 2012 - 2014 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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SELF DECLARATION

I, Mr Rohit Kumar Singh, Roll No. 212ME1284, student of M. Tech (2012-14), Machine Design and Analysis at Department of Mechanical Engineering, National Institute of Technology Rourkela do hereby declare that I have not adopted any kind of unfair means and carried out the research work reported in this thesis work ethically to the best of my knowledge. If adoption of any kind of unfair means is found in this thesis work at a later stage, then appropriate action can be taken against me including withdrawal of this thesis work.

NIT Rourkela

02 June 2014

Rohit Kumar Singh

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Rohit Kumar Singh

Abstract

Laminated composite shell panels are increasingly used in aeronautical, marine and mechanical industries as well as in other fields of modern technology because of its advance mechanical properties. It is well known that the composites have high strength to weight ratio and stiffness to weight ratio as compared to any conventional materials like concrete, metal, and wood. As the uses of the composites in different industries have increased which leads to their analysis through mathematical, experimental and/or simulation based model for accurate design and subsequent manufacturing. The structural components of aircraft, launch vehicle and missiles are subjected to various types of combined loading and exposed to large acoustic, vibration, inertia excitation during their service life. In addition to that the structural components are also exposed to elevated thermal environment due to the aerodynamic heating. This often changes the original geometry of the panel due to excess deformation and the structural performances reduce. In this work, thermal buckling behaviour of laminated composite curved panel embedded with shape memory alloy fibre (SMA) is investigated. The material properties of composite laminate and SMA fibres are taken as temperature dependent. The mathematical model is developed based on higher order shear deformation theory (HSDT) to count the exact flexure of the laminate. The buckling behaviour is evaluated based on the Green-Lagrange strain-displacement equations for in-plane strains to account the large deflections under uniform temperature loading. The nonlinear material behaviour of shape memory alloy is introduced through a marching technique. The responses are obtained using variational principle in conjunction with suitable isoparametric finite element modelling based on the MATLAB code. In addition to that, a simulation model is also being developed in ANSYS environment using ANSYS parametric design language code for laminated composites curved panel and the corresponding responses are compared with those available literatures. The effects of layup sequence, thickness ratio, ply angle, support conditions and temperature dependence material properties on thermal buckling load is obtained and discussed.

Keywords: Laminated panel, HSDT, Green-Lagrange nonlinearity, FEM, Thermal buckling, ANSYS, APDL code, MATLAB.

Contents

List of Symbols

Most of the symbols are defined as they occur in the thesis. Some of most common symbols, which are used repeatedly, are listed below:

x, y, z	Co-ordinate axis
u, v and w	Displacements corresponding to x, y and z directions, respectively
u_0, v_0 and w_0	In-plane and transverse displacements of a point (x, y) on the mid-plane
θ_x, and θ_y	Rotations of normal to the mid-plane
ϕ_x, ϕ_y, ψ_x and ψ_y	Higher order terms of Taylor series expansion
R_x, R_y	Principal radii of the curvatures of the shell panel
E_1, E_2 and E_3	Young's modulus
G_{12}, G_{23} and G_{13}	Shear modulus
υ_{12}, υ_{23} and υ_{13}	Poison's ratios
a, b and h	Length, width and thickness of the shell panel
$\{\varepsilon_l\}$	Linear strain vectors
$\{\sigma\}$	Stress vector at mid-plane
$\{\delta\}$	Displacement vector
$[\bar{Q}]$	Transformed reduced elastic constant
$[B_l]$	Linear strain displacement matrix
$[D]$	Rigidity matrix

$\left[K_S\right]$	Linear stiffness matrix
$\left[K_G\right]$	Geometric stiffness matrix
F	Global force vector
$U_{\text{S.E.}}$	Strain energy
$\left[T\right]$	Function of thickness co-ordinate
α	Thermal expansion co-efficient
ρ	Density of the material
T	Kinetic energy
W	Work done
a/h	Thickness ratio
R/a	Curvature ratio
E_1/E_2	Modular ratio
λ_{cr}	Non-dimensional buckling temperature

Subscript

l	Linear
2	Anti- Symmetric
S	Symmetric
i	Node number

Abbreviation

CLPT	Classical laminate plate theory
FSDT	First order shear deformation theory
HSDT	Higher order shear deformation theory
APDL	ANSYS parametric design language
SSSS	All edges simply supported
CCCC	All edges clamped
HHHH	All edges hinged
Eq.	Equation
GPa	Giga Pascal

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CHAPTER 1

INTRODUCTION

1.1 Overview

Laminated composite shell panel are extensively used in many industries, building, bridges and structures such as boat hulls, racing car body and storage tank and some of the advance application of laminated composite shell panel is used in naval and space projects. Composites are made up of two constituent elements matrix and fibre. Matrix provides compliant support for the reinforcement and distributes the load evenly in all direction, as it surrounds the fibre whose function is to bear the load. Composite materials has a following property which make it attractive to the researcher such as its volume to its weight ratio, low co-efficient of thermal expansion, outstanding elastic properties and good corrosion and chemicals resistant. Laminated composite panel is assemblies of layers of fibrous composite material which can be used for specific purpose. Laminated composites panel are extremely lightweight with respect to conventional materials like concrete, metal, and wood. Because of above properties it is being used in like aerospace, marine, modern automobile and modern structure. Shell panel is a curved thin and thick structure having single or multilayer of anisotropic or orthotropic materials subjected to different loads. The shell panel can be classified according to its curvatures such as flat panel (as its both the radius is zero), cylindrical (one radius is zero), spherical (where both the radius is equal), conical (where one radius is zero and another varies linearly with the axial length). The shell panel has considerably higher membrane stiffness than that of the bending stiffness that's why it can withstand a large value of membrane strain energy without large deformations. The external skins of aircraft/spacecraft/automobile are having panel type of geometry and made of the thin laminated composites.

If a shell is subjected to compressive load, it will be store as strain energy and on further increases of the load leads to bending in the structure, progressively leads to a buckling failure of the structure. Hence, the buckling plays an important role in the design and analysis of the structures. Basically two types of buckling occur in structural member namely, eigenvalue/ bifurcation buckling and non-linear/ limit point buckling. The bifurcation buckling is a form instability in which there is a sudden change of shape of the structure due to the axial compressive/tensile load. However, in the limit point buckling there is no sudden change of shape but it deviates from the primary equilibrium path after reaching the critical load i.e., known as “snap through”. Buckling doesn’t mean failure of structural component as it is capable enough bear more load even after the point of buckling.

To increases the structural performance of the structure against thermal loading, conventional structural undergoes structural stiffening and constraining, but this conventional method leads to increase in weight and temperature rise, it is substantially high enough to create enough problem than that of its advantages. So to solve such problems smart material have been introduced which contains actuator, sensor and microprocessor capability like pH-sensitive polymers, piezoelectric material (produces electricity in response to stress), functionally graded material (material changes property in thickness direction), shape memory alloy (material property changes with the variation with temperature) and photomechanical material (material-change shape under exposed to light). These new materials have the property that unable designer to change the external as well as internal condition which leads to advancement in the structural performance. The smart material has the great affinity to change shape, frequency, stiffness, buckling, damping, and other mechanical parameter with respect to change in electrical field, temperature or magnetic field. In the present investigation SMA embedded composite shell panel is used to regulate reaction of the shell panel exposed to thermal load.

SMA has been researched for last 30 years due to its attractive functional properties. The shape memory effect i.e. the property of recovering the deformation when exposed to heating up to a certain elevated temperature, was first discovered in an alloy of Cadmium (52.5%) and Gold (47.5%) by Otsuka and Wayman in 1932. Again this behaviour was observed in Cu-Zn in 1938 and later on this property was discovered and enriched by United States naval ordnance laboratory in 1962 in alloy of nickel (48%)-titanium(52%) and commercialized it under the trade name of Nitinol. SMAs have the better competency for engineering application because it is highly ductile at low temperature shape, recovery capability is 10% strain which is high enough, large corrosion resistance and biomedical application.

SMA is a specific type of material that has the ability to regain its original shape when heated up to a certain elevated temperature and if it is deformed up to a certain limit. SMA exhibits in two stable state of lattice structure namely austenite and martensite. At the higher temperature austenite phase is more stable state and has the symmetrical cubic crystal lattice structure. And at the lower temperature, martensite phase is stable state and has the less symmetric lattice structure of a monoclinic. Both the state can be seen in the Fig. 1.1(webdocs.cs.ualberta.ca). In addition, martensite phase in SMA exists in two state as twinned and de-twinned (M+, M-) state.

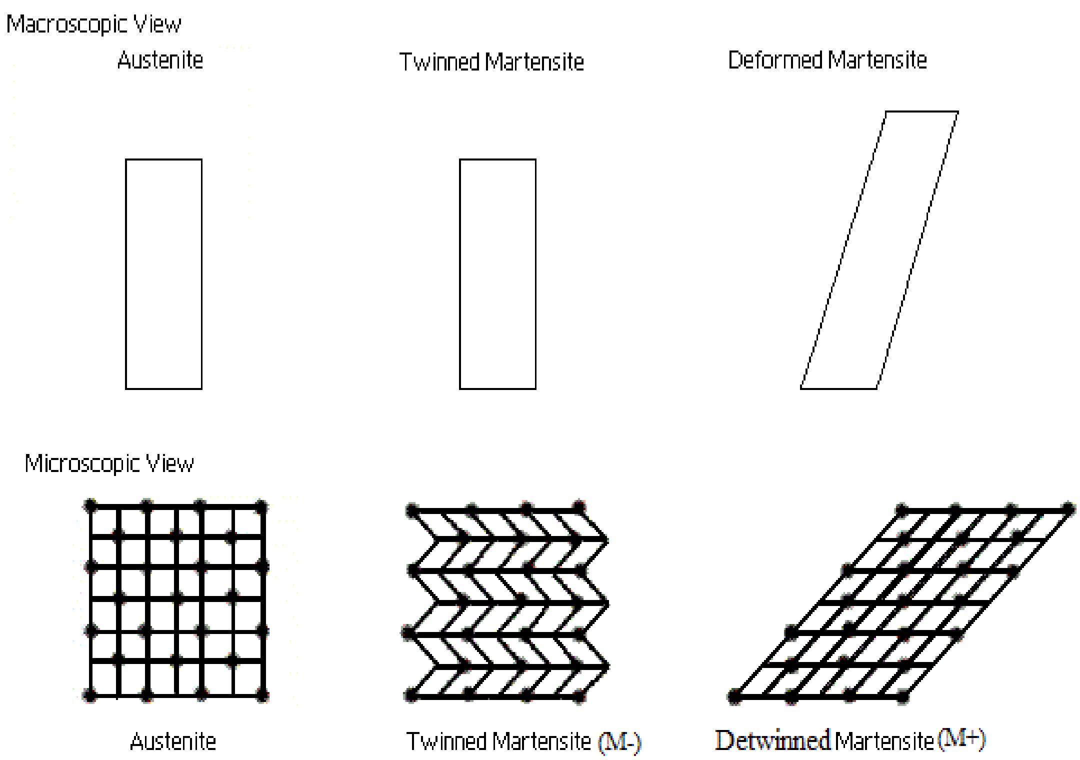

Fig. 1.1 SMA Lattice Structure at Different Phase

The SMA transformation from austenite phase to the self-accommodation martensite twinned phase when cooled under no mechanical load and there is no transformation with the change in shape. Therefore, deformation can easily be achieved in the martensite state because the boundary layers can easily deform due to applied load.

A tensile load is applied to SMA which is in the martensite state i.e. martensite twinned configuration which will transform, at a critical load, to M+ or M- variants depending on the direction of load applied. The SMA will remain in M+ phase with the given deformation and further when the load is removed and SMA is heated up of the martensite phase, the "reverse transformation" occurs in which the lattice structure returns to austenite phase, which may lead to the recovery of any deformation as shown in Fig. 1.2.

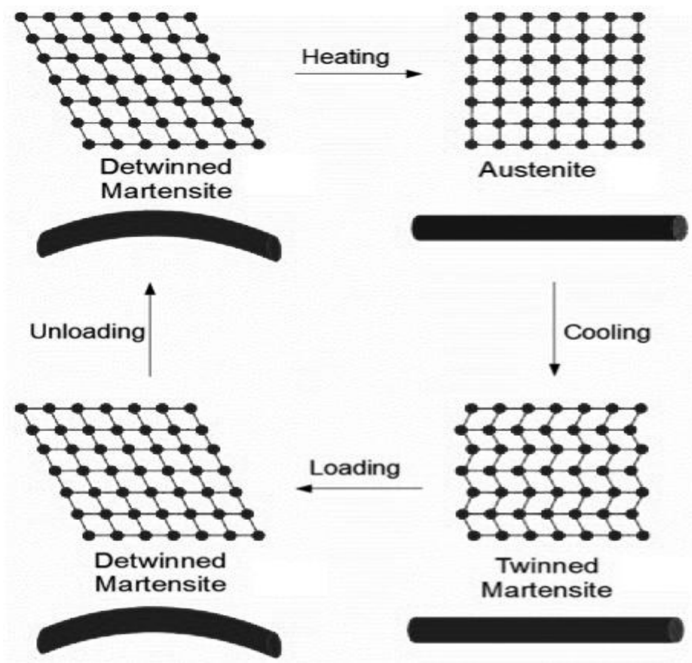

Fig. 1.2: Phase Transformation in SMA and Different Lattice Structure

This phase transformation property of SMA is called "shape memory effect" (SME). The SMA has four characteristics temperature during its reverse phase transformation as shown in Fig. 1.3. Martensite start (M_s) is the temperature at which the SMA being transformation from austenite to martensite; martensite finish (M_f) is the temperature at which the SMA becomes fully martensite and the transformation is completed; austenite start (A_s) represents the temperature at which "reversal transformation" begin; and austenite finish (A_f) represent the temperature at which the SMA is in a fully austenite phase.

The SMA shows transformation psedudoelastic behaviour, which shows the recovery of strains on the removal of loads. Pseudoelastic behaviour of the SMA is eye-catching property by virtue of which it can be used as an energy dissipating mechanism, and capable to produce hysteretic damping of structures.

Nitinol work in the temperature range of -50 ºC (-122 ºF) to 166ºC (330ºF), if the Ni content is changed from 54.4% to 56.5% by weight and is fully annealed, the alloys possess phase transformation directly from austenite to martensite. By virtue of this property a very small strain recovery of strain and hysteresis takes place that is an attractive property by virtue of which it can be used as a sensor.

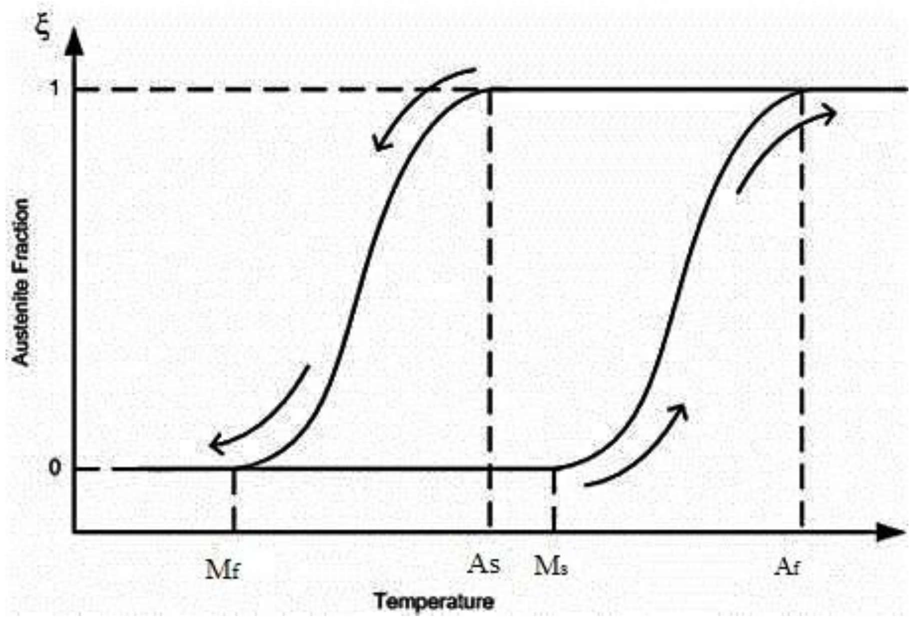

Fig. 1.3 Shows the phase transformation and different phase temperature

Based on the different facts and considerations, it is comprehended that a better generalized nonlinear shell panel model is to be generated to predict the thermal buckling temperature of composite shell panel embedded with and without SMA fibre and more accurately especially in that case where deflection is very large. Model should be developed such that it would be capable enough to consider the effect of geometrical nonlinearity and the materials nonlinearity in Shape Memory Alloy due to uniform temperature increment for highly nonlinear problems. A mathematical model is to be developed in Green-Lagrange sense based on HSDT, to understand the realistic nature of the nonlinear structural response of a curved panel embedded with and without SMA fibre. By above assumption and consideration the design and analysis of structure is more reliable in practical application. As it can be said that the model developed would be capable enough to resemble the real time laminated shell model but, this problem is not only interesting but also challenging in many front. Hence, while modelling of these structures major aspects of the designer is to predict the critical buckling temperature parameter. The property and orientation should be studied properly and appropriately judged as that helps in solving the operation and problem. The previous parametric study helps us to find the result by considering the consequences of loading and limiting conditions. Problem should be studied thoroughly before the analysis and modelling.

1.2 Introduction of Finite Element Method and ANSYS

With the advancement in technology, the design process is too close to precision, so the finite element method (FEM) is used widely and capable to draw complicated structure and this is very trusted tool for designing of any shape and structure. It plays an important role in predicting the responses of various products, parts, assemblies and subassemblies. Nowadays, FEM is extensively used by all advanced industries which save their huge time of prototyping with reducing the cost due to physical test and increases the innovation at a faster and more accurate way. There are many optimized finite element analysis (FEA) tools are available in the market and ANSYS is one of them which is acceptable to many industries and analysts.

Nowadays, ANSYS is being used in different engineering fields such as power generation, electronic devices, transportation, and household appliances as well as to analyse the vehicle simulation and in aerospace industries. ANSYS gradually entered into a number of fields making it convenient for fatigue analysis, nuclear power plant and medical applications. ANSYS is also very useful in electro thermal analysis of switching elements of a super conductor, ion projection lithography, detuning of an HF oscillator.

Here the buckling analysis is done by taking shell element SHELL281 from the ANSYS library shown in Figure 1.4. It is an eight-noded linear shell element which has six degrees of freedom at each node with possible translation motion in x, y, z direction and rotation about x, y, z axis.

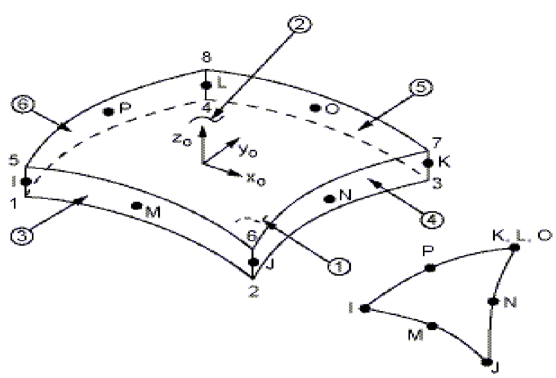

Figure 1.4 SHELL281 Geometry

x_o = Element x-axis if element orientation is not provided.

x = Element x-axis if element orientation is provided.

1.3 Motivation of the Present Work

The laminated composite shell panels are of great attention to the designers because of its efficient lightweight to strength ratio, high-impact strength, dimensional stability, corrosion resistance and low thermal conductivity. In the past few years, use of composite structures has increased a lot especially in aeronautical/aerospace engineering which forced the engineers for its analysis. These structural components are undergone to various types of combined loading and goes through high temperature during their service period, which may leads to change in the shape of the geometry of structure. The changes in panel geometry and the interaction with loading condition affect the buckling responses greatly.

In order to achieve the light weight structures for stringent demand of weight reduction in the advanced engineering structures to conserve energy, the laminated composites consisting of multiple layers are extensively employed and their usage will continue to grow as structural members. It is also important to mention that, these laminated composites are weak in shear and highly flexible in nature as compared to any other metallic plate/shell. To obtain the accurate prediction of responses of laminated composites, it is necessary and essential requirement that the displacement model must be capable to take care of the consequence of shear deformation. In this regard a HSDT is most desirable. The geometry of the shell panel alters and stiffness matrix associated with the material are no more linear due to excess deformations and this effects has to be appropriately considered in the analysis. Buckling of structures have been received a considerable attention not only due to their wide range application, but also the challenging problems with interesting behaviour. In most of the literature, the geometry matrix associated in buckling is modelled taking into account for the non-linearity in the von-Karman sense. But the nonlinearity in von-Karman sense may not be appropriate enough for the realistic prediction of their responses. Since the existing studies considering all these aspects are not sufficient enough to predict the accurate structural responses so, there is a need of a better general model for the more accurate estimation of the behaviour of laminated shell panels.

1.4 Objective and Scope of the Present Thesis

This study aims to develop a general mathematical model for laminated composite curved panel under uniform temperature loading based on the HSDT displacement field model. The Green-Lagrange type of strain displacement relations are employed to account the geometrical distortion. A suitable finite element model is approached to discretise the present model and responses are obtained, subsequently. The effect of different types of panel geometries (cylindrical, spherical and flat) and other geometrical parameters (aspect ratio, thickness ratio, curvature ratio, support condition and stacking sequence) on the thermal buckling responses of the laminated composites shell panel embedded with/without SMA, are analysed and discussed. Point-wise details of the present study are listed below:

- A mathematical model is developed based on the HSDT mid-plane kinematics with the incorporation of Green-Lagrange strain terms.
- The finite element solutions are provided by using a nine noded quadrilateral element with nine degrees of freedom per node.
- A computer code has been developed in MATLAB environment to obtain the desired responses.
- The present study also extended for composite shell panel modelled through APDL code in ANSYS 13.0 environment.
- Finally, the parametric study of laminated composite panel embedded with/without SMA has been done by using APDL model and developed HSDT model.

1.5 Organization of the Thesis

The overview and motivation of the present work followed by the objectives and scope of the present thesis are discussed in this chapter. The background and state of the art of the present problem by various investigators related to the scope of the present area of interest are addresses in this chapter. This chapter divided into five different sections, the first section, a basic introduction about problem and theories used in past. In the section two, some important contributions for thermal buckling behaviour of laminated composite structures are discussed. In the section three, a brief introduction of finite element method and finite element analysis software ANSYS is presented. The motivation of the present work is discussed in fourth and in fifth objective and scope of present work is incorporated. Some critical observations are discussed in the final section. The remaining part of the thesis are organised in the following fashion.

In chapter 2, brief introduction of the previous publish literature has been presented along with their theory and method adopted for the analysis by the authors. The chapter is subdivided in two parts consisting of thermal buckling analysis of composite shell panel without SMA and thermal buckling analysis of composite shell panel embedded with SMA.

In chapter 3, a general mathematical formulation for the thermal buckling of laminated composite panel, by modelling in the framework of the HSDT under the uniform temperature distribution. The Green-Lagrange type strain displacement relations are considered to account the geometrical nonlinearity arising in the shell panel due to excess deformation. The steps of various energy calculations, governing equation and solution steps are discussed. Subsequently, the boundary condition and computational investigation are discussed.

Chapter 4, illustrate the thermal buckling responses of laminated composite panels for various panel geometries such as cylindrical, spherical and flat panel are discussed. Detailed parametric studies of material and geometrical parameters are also discussed.

Enhancement of thermal buckling of laminated composite panels for different panel geometries and the influence of geometrical and material properties on the panel responses under the influence of Shape Memory Alloy are discussed in Chapter 5.

Chapter 6 summarizes the whole work and it contains the concluding remarks based on the present study and the future scope of the work.

Some important books and publications referred during the present study have been listed in the References section.

In order to achieve the objective and scope of the present work discussed above in this chapter, there is need to know the state of art of the problem for that a detailed review of earlier work done in the same field have been discussed thoroughly in the next chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Flat/curved composite panels and their combination are used at the place where the light weight places an important role like aerospace, marine, mechanical and modern automotives. The curved geometry and/or meridional discontinuity of the panel due to the the joints increases the geometrical complexity and adversely affect the stability. Structures are very often subjected to buckling and post-buckling. They are also subjected to high thermal load during their working period. Due to the boundary constraints, the development compressive thermal membrane state of stress may lead to structural instability/buckling of the structure.

In general, laminated structural component are under combined effect of aerodynamic, mechanical and thermal loading condition during their operational life for which a significant difference exist between their deformed and undeformed shape and the linear strain displacement relation are not only able to explain the state variable. The strain displacement relation and the displacement relations and the displacement model should be sufficient enough to accomplish the excess thermal deformation and/or the large amplitude vibration of the structure for an accurate prediction. The laminated structure are highly flexible in nature and when working under various loading condition as discussed then the geometrical nonlinearity in von-Karman sense is unrealistic in nature for the mathematical modelling purpose. Hence, there is a need of rigorous study of different structural behavior using a better nonlinear model to characterise the post-buckling characteristic in details.

A lot of literatures are presented on thermal buckling of laminated composite shell panel embedded with and without SMA fibre by taking the nonlinearity in von-Karman sense in the framework of various classical and shear deformation theories such as CLT, FSDT, LWT and HSDT. Relatively very less number of studies is reported on the buckling temperature of laminated shell panel with and without SMA by taking the nonlinearity in von-Karman sense or Green-Lagrangian sense in the framework of the HSDT. In the following section of this chapter a brief contextual of various type of problem and comprehensive reviews of existing literatures is discussed. The review of literature is carried out for buckling for composite panel with and without SMA fibre for the present investigation. In continuation buckling characteristic due to thermal load is discussed in this addition. Finally, a heading is devoted for the critical observations obtained from the discussed literature.

2.2 Thermal Buckling Analysis

The buckling is nothing but the geometrical shape change of structural component and is usually nonlinear in nature. A numerous analyses have been reported in the literature on the buckling by taking the geometric matrix and nonlinear stiffness matrices in von-Karman sense based in various theories such as classical laminated theory and the shear deformation theories. Lots of literatures are there on buckling of composite shell panel due to thermal loadings. A few of them are discussed here.

2.2.1 Thermal buckling analysis of laminated composite without SAM fibre

Matsunaga (2005) predict the buckling stress and natural frequencies of laminated composite beams based on the power series expansion. Sahin (2005) studied buckling behaviour of symmetric and anti-symmetric cross-ply laminated hybrid composite plates with a hole using FEM based on the FSDT model. Shariyat (2007) investigated post buckling behavior of laminated imperfect plates based on the layer wise theory and von Karman nonlinear strain–displacement equations. The composite material properties are calculated based on the temperature effect for the computation. Chen and Chen (1991) reported thermal post-buckling behavior of laminated composite plate using Hermitian polynomials based on the temperature dependent material properties. The elastic properties of the medium are assumed to be temperature dependent. Shukla and Nath (2001) computed the buckling and post-buckling load parameters of angle-ply laminated plates analytically under thermo-mechanical loading. The non- linearity in geometry matrix due to excess thermal/mechanical deformation is taken in von-Karman sense in the framework of the FSDT. Thankam et.al. (2003) investigated thermal buckling and post-buckling behaviour of cross ply and angle-ply laminates.

Modified feasible direction method has been used by Topal and Uzman (2008) for thermal buckling load optimization of laminated flat/cylindrical panels to maximize the critical temperature capacity of laminated structures. Shiau (2010) reported critical buckling load and modes of two different laminations (cross-ply and angle-ply). Shen (1997) analyzed the buckling and post-buckling behavior of laminated composite plate based on HSDT and von-Karman nonlinear kinematics. Shen (2001) reported buckling and post buckling temperature and load deflection curve of laminated composite plates based on Reddy’s (2003) third composite and sandwich panels based on the global higher order shear deformation theory by solving order shear deformation theory. Ovesy et. al. (2009) predicted thermal buckling load used the concept of principle of minimum potential energy and an eigen-value analysis is subsequently carried out along with implementing the higher order semi-analytical finite strip method and concluded that classical laminated plate theory (CLPT) predict the buckling load more accurately. Singh and Shukla (2012) studied stability of orthotropic cross ply laminated composite plates subjected to thermal and mechanical. Shen (2001) employed the perturbation technique and thermal buckling and post buckling temperature load deflection curved is drawn by an iterative numerical procedure by the FSDT model.

Thermal buckling responses of anti-symmetric angle-ply laminated plate are obtained by Chang and Leu (1991) based on a higher-order displacement field and solved using 3-D elasticity solution. Lee (1997) studied thermal buckling behaviour of laminated composite plates based on the layerwise theory by using FEM steps. Vosoughi et al. (2012) presented thermal buckling and post-buckling load parameters of laminated composite beams. The buckling behaviour of composite plates is analysed by Jameel et al. (2012) under thermal and mechanical loading. Buckling behaviour of laminated composite plate is studied by Fazzolari et al. (2013) taking non- linearity in geometry matrix due to excess thermal/mechanical deformation in von-Karman sense in the framework of the HSDT. Shadmehri et al. (2013) reported stability behaviour of conical composite shells subjected to axial compression load using linear strain–displacement relations in the framework of the FSDT. Kheirikhah et al. (2012) studied bi-axial buckling behaviour of laminated composite and sandwich plates using the von-Karman kinematic non-linearity in the framework of the HSDT. Nali et al. (2011) obtained buckling responses of laminated plates using von-Karman nonlinear kinematic in the framework of thin plate theory. Singh et al. (2013) reported the buckling behaviour of laminated composite plates subjected to thermal and mechanical loading using mesh-less collocation method in the framework of the HSDT.

In addition to the above, some researcher have also analysed the buckling behaviour of laminated structures under mechanical loading using the same type of geometrical nonlinearity and in-plane mechanical loading. Khdeir and Librescue (1988) investigated the buckling and the free vibration responses of symmetric cross-ply laminated elastic plates based on the HSDT mid-plane kinematics. The buckling behaviour of cross-ply laminated conical shell panels subjected to axial compression is studied by Abediokhchi et al. (2013) in the framework of the classical shell theory. Komur et al. (2010) obtained buckling responses of laminated composite plates with an elliptical/circular cut-out using FEM and governing equations are solved using Newton–Raphson method. Seifi et al. (2012) reported buckling responses of composite annular plates under uniform internal and external radial edge loads in the framework of the CLPT. Khalili et al. (2013) obtained buckling load parameters of laminated rectangular plate on Pasternak foundation using the Lindstedt–Poincare perturbation technique. Tang and Wang (2011) studied buckling behaviour of symmetrically laminated rectangular plates with in-plane compressive loadings using the Rayleigh–Ritz method in the framework of the CLPT.

2.2.2 Thermal buckling and post-buckling analysis of laminated composite embedded with SAM fibre

The buckling and post-buckling strength of laminated structure can be enhanced comprehensive by employing SMA in thermal environment due to the inherent actuation properties in the Chapter 1. Many studies are reported in the literature, to exploit the actual strength of SMA against different type geometric and environmental condition. Lee and choi (1999) developed an analytical formula to predict the buckling load and enhancement of post-buckling strength of composite beams by using SMA. Poon et al. (2004) developed a simple equation to study the effect of size on the actuation of SMA wire actuator inside the hybrid composites numerically. Naseer et al. (2005) studied the plastic behaviour of SMA due to very high strain rate subjected to compressive load experimentally. Burton et al. (2006) studies numerically the healing of cracks in the laminated composite structural components embedded with SMA wires by varying the temperature based on FEM. Thomson and loughlam (1997 and 2001) analysed the adaptive post-buckling responses of laminated composite carbon fibres embedded with prestrain SMA fibres and predicted the enhancement of post-bucking strength of laminated composites and non-uniform temperature profile within the laminates using FEM under uniaxial load.

The SMA can be used as a smart material for the futuristic structures, where light weight and controllability are main factor of considerations. Based on that, few researched have studies on the feasibility of SMA application especially in aerospace structures. Loughlan et al. (2002) investigated the enhancement of buckling load and suppression post-buckling deflection based on various experimental studies and discussed the feasibility of application of SMA in aerospace structure as a smart material. Hartl and Lagoudas (2007) published a detailed review article on the SMA application in aerospace structures as a multifunctional material.

Park et. al. (2005) studied the consequence of SMA on critical temperature, thermal post-buckling deflection, natural frequency and critical dynamic pressure of SMA composite plate subjected to aerodynamics and thermal loading. Kumar and Singh (2009) reported thermal buckling and post-buckling behaviour of laminated composite flat panel embedded with SMA fibre using Newton-Raphson method under uniform thermal load.

Ganilova and Cartmell (2009) studied the vibration behaviour of SMA by developing mathematical model by means of WKB- Galerkin method. Here damping and stiffness are considered time dependent. Burton et al.(2004) studied the crack healing behaviour under the influence of SMA while simulation is done in a ABAQUS. Panda and Singh (2013) investigated the vibrational behaviour of laminated composite panel embedded with SMA fibre by taking the geometrical nonlinearily in Green-Lagrange sense. Qiao et al. (2013) studied the behaviour of shape memory polymer composites (SMPCs) and calculated the postbuckling temperature using FEM. Muhammad et al. (2000) observed the buckling and postbuckling behaviour of SMA under the varying value of slenderness ratio (L/k) and compared with some other materials.

Loughlan et al. (2002) investigated the consequence of temperature on buckling behaviour of composite embedded with SMA. Thompson and Louhglan (2001) shown the effect of restoration force on post-buckling response as the material is exposed to elevated temperature and the result is compared with that of conventional material and structure. Panda and Singh (2011) studied the thermal post-buckling strength of doubly curve in Green-Lagrangian sense in framework of HSDT and effect of various geometrical parameters. Lee et al. (2005) studied the behaviour of ferromagnetic shape memory alloy (FSMA) by using ANSYS code result is compared with that of SMA and is illustrated reliability of FSMA. Choi and Toi (2009) studied the 3-D superelastic behaviors of SMA devices and calculated the various properties.

CHAPTER 3

GENERAL MATHEMATICAL FORMULATION

3.1 Introduction

Now days, high performance laminated structure and their components are designed and used in many places like marine, space and many more. These structures are more reliable and have more a load bearing capacity as they often account varying load such as structural, thermal, large amplitude flexural vibration. Hence, the demand of detailed study of such structure under above mentioned condition is increasing very rapidly and significantly.

In general, nonlinearity in the structural analysis is of two type geometrical nonlinearity and geometrical nonlinearity. As we all know that the total deformation occurred in the material system is a sum of translational, rotation and distortion components and if the structure undergoes severe nonlinearity then only two other component rather than distortion component play important role to derive the actual strain- displacement relation. The laminated composite structural components are highly flexible with respect to metallic component so higher order nonlinear terms and large deformation terms in mathematical modelling are important for an accurate analysis. As per detailed literature study in the previous chapter clearly shows that many analyses have been done for many nonlinear problems, but no study have been done using geometrical nonlinearity in Green-Lagrange sense in the framework of HSDT for laminated composite shell panel embedded with and without SMA fibre.

In this chapter general formulation of curved panel is developed on the basis of basic assumptions. A FE model is produced to discretise the present model though chosen displacement field. The geometrical nonlinearity in green-Lagrange sense and the material nonlinearity for SMAs stress-strain relation are considered for the formulation and discussed in this chapter. Finally, to solve the algebraic equations a direct iterative procedure is adopted.

3.2 Assumptions

The following assumptions are taken for the mathematical formulation:

- The middle panel of the curved panel is taken as the reference plane.
- Every layer is assumed to be homogeneous and orthotropic whereas the layer bonded together well.
- Two dimensional approach has been adopted to model a three dimensional behaviour of shell.
- A uniform temperature field is taken for the analysis in the thickness direction.
- The composite material properties are considered temperature dependent and independent whereas the SMA material properties are taken temperature dependent.

3.3 Geometry of the Shell

Using the orthogonal curvilinear coordinates system such that X and Y curves are line of curvature on mid surface (Z=0), and Z- curvature is straight line perpendicular to the surface Z=0. The position vectors are described (Reddy 2004) to show the position of any arbitrary point on the deformed shell geometry. The position vector of the point (X,Y,0) on the middle surface denoted by r, and the position vector of an arbitrary point (X,Y,Z) is denoted by R. SMA layer is improvised in composite plate above and below the middle layer of the plate.

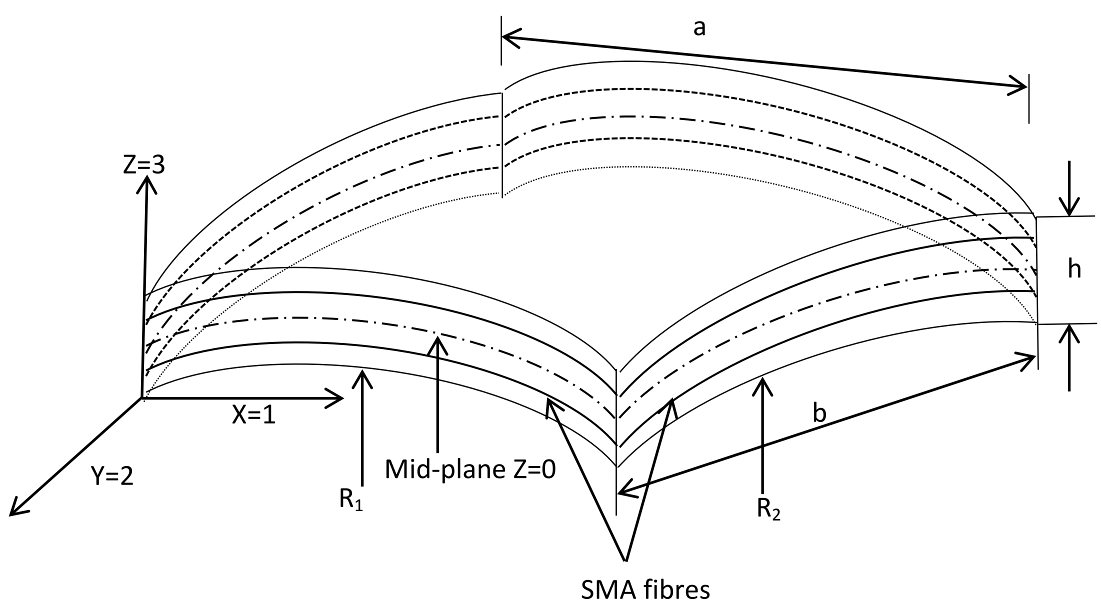

Fig 3.1 Laminated composite shell panel embedded with SMA fibre

3.4 Displacement Field

A shell panel of length a, width b, and height h is composed of finite orthotropic layer of uniform thickness. The the principal radii of curvatures are symbolised with R_1 and R_2. The Z_k and Z_{k-1} is the top and bottom Z- coordinate of the k^{th} lamina. The following displacement field for the laminated shell panel based on the HSDT suggested by Reddy and Liu (1985) is taken to derive the mathematical model.

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) + z^2\phi_x(x,y,t) + z^3\psi_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) + z^2\phi_y(x,y,t) + z^3\psi_y(x,y,t) \\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned}
\tag{3.1}
$$

where t is the time, (u,v,w)are the displacement along the (x,y,z) coordinate. (u_0,v_0,w_0) are the displacement of a point on the mid-plane and θ_x, θ_y and θ_z are the rotation at z=0 of normal to the mid-plane with respect to the y and x axes, respectively. ϕ_x, ϕ_y, ψ_x and ψ_y are the higher order terms of the Taylor series expansion defined at the mid-plane.

3.5 Strain-Displacement Relation

The linear green-Lagrange strain displacement relations are adopted for the laminated flat panel which are expressed as (Reddy 2004):

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial u}{\partial x} + \dfrac{w}{R_x}\right) \\ \left(\dfrac{\partial v}{\partial y} + \dfrac{w}{R_y}\right) \\ \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) \\ \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} - \dfrac{u}{R_x}\right) \\ \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} - \dfrac{v}{R_y}\right) \end{Bmatrix}
\tag{3.2}
$$

or, $\{\varepsilon\} = \{\varepsilon_L\}$

where $\{\varepsilon_L\}$ is the linear strain vectors, respectively.

Substituting the Eq. (3.1) value into Eq. (3.2) the strain-displacement relation is stated for laminated shell panel as

$$\{\varepsilon_L\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \varepsilon_{xy}{}^0 \\ \varepsilon_{xz}{}^0 \\ \varepsilon_{yz}{}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x{}^1 \\ k_y{}^1 \\ k_{xy}{}^1 \\ k_{xz}{}^1 \\ k_{yz}{}^1 \end{Bmatrix} + z^2 \begin{Bmatrix} k_x{}^2 \\ k_y{}^2 \\ k_{xy}{}^2 \\ k_{xz}{}^2 \\ k_{yz}{}^2 \end{Bmatrix} + z^3 \begin{Bmatrix} k_x{}^3 \\ k_y{}^3 \\ k_{xy}{}^3 \\ k_{xz}{}^3 \\ k_{yz}{}^3 \end{Bmatrix} \tag{3.3}$$

From above

$$\{\varepsilon\} = [H]_L \{\bar{\varepsilon}_L\} \tag{3.4}$$

where, $\{\bar{\varepsilon}_l\}$ is the linear mid-plane strain vectors and $[H]_L$ is the linear thickness coordinate matrix.

The mid plane linear strain vector is given by

$$\{\bar{\varepsilon}_l\} = \left[\varepsilon_x{}^0\,\varepsilon_y{}^0\,\varepsilon_{xy}{}^0\,\varepsilon_{xz}{}^0\,\varepsilon_{yz}{}^0\;k_x{}^1\,k_y{}^1\,k_{xy}{}^1\,k_{xz}{}^1\,k_{yz}{}^1\,k_x{}^2\,k_y{}^2\,k_{xy}{}^2\,k_{xz}{}^2\,k_{yz}{}^2\,k_x{}^3\,k_y{}^3\,k_{xy}{}^3\,k_{xz}{}^3\,k_{yz}{}^3\right]^T$$

where, the terms containing superscripts 0, 1, 2, 3 are the bending, curvature and higher order terms.

3.6 Material Properties of Composite Embedded with SMA Fibre

The principal material directions of the shell panel say, 1, 2 and 3. The material properties of laminated composite shell panel embedded with SMA fibre are expressed as

$$E_{1e}=E_1V_m+E_sV_s \tag{3.5}$$

$$E_{2e}=E_2E_s/\,(E_2V_s+E_sV_m) \tag{3.6}$$

$$G_{12e}=G_{12}G_s/\,(G_{12}G_s+G_sV_m) \tag{3.7}$$

$$G_{23e}=G_{23}V_m+G_sV_s \tag{3.8}$$

$\nu_{12e}=\nu_{12m}V_m+\nu_s V_s$ (3.9)

$\alpha_{1e}=(E_{1m}\alpha_{1m}V_m+E_s\alpha_s V_s)/E_1$ (3.10)

$\alpha_{2e}=\alpha_{2m}V_m+\alpha_s V_s$ (3.11)

where subscript 'm' and 's' indicates the composite matrix and SMA fibres, respectively. Here E, G, ν and α are the Young's modulus, shear modulus, Poisson's ratio and thermal expansion co-efficient respectively. V_m and V_s are the volume fraction of the composite matrix and SMA fibre, respectively.

3.7 Constitutive Relation

For orthotropic lamina the stress-strain relations for k^{th} layer of SMA embedded composite matrix in material coordinate axes under constant temperature field are expressed as (Park et al. 2004):

$$\{\sigma\}^k = \left[\bar{Q}\right]^k \{\varepsilon\}^k + \{\sigma_{\Delta r}\}^k \{\sigma\}^k V_s^k - \left(\left[\bar{Q}\right]_m^k \{\alpha\}_m V_m\right)^k \Delta T \tag{3.12}$$

where, $\{\sigma\}^k = \{\sigma_1 \ \ \sigma_2 \ \ \sigma_6 \ \ \sigma_5 \ \ \sigma_4\}^T$ is the total stress vector and $\{\sigma_{\Delta r}\}$ is the recovery stress produced in SMA fibre due to the temperature change (ΔT) and $\{\varepsilon\}^k = \{\varepsilon_1 \ \ \varepsilon_2 \ \ \varepsilon_6 \ \ \varepsilon_5 \ \ \varepsilon_4\}^T$ is the strain vector, in the k^{th} layer. In addition, $\left[\bar{Q}\right]^k$, $\left[\bar{Q}\right]_m^k$ and $[\alpha]_m^k = \{\alpha_{1m} \ \ \alpha_{2m} \ \ 2\alpha_{12m}\}^T$ are the transferred reduced stiffness matrix of SMA embedded lamina, transferred reduced stiffness matrix of composite matrix and the transformed thermal expansion coefficient vector of the k^{th} layer, respectively. ΔT is the change in temperature.

$$\begin{Bmatrix} \{N_{\Delta t}\} \\ \{M_{\Delta t}\} \\ \{P_{\Delta t}\} \end{Bmatrix} - \begin{Bmatrix} \{N_{\Delta r}\} \\ \{M_{\Delta r}\} \\ \{P_{\Delta r}\} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{\sigma_{\Delta r}\right\}^k V_s^k - \left[\bar{Q}\right]_m^k \begin{Bmatrix} \alpha_{1m} \\ \alpha_{2m} \\ 2\alpha_{12m} \end{Bmatrix}^k (1, z, z^3) V_m \Delta T dz \tag{3.13}$$

Thermal in-plane generated force can be obtained by integrating the equation (3.12) in the thickness direction and can be represented in matrix form as:

where $\{N_{\Delta t}\}$, $\{M_{\Delta t}\}$ and $\{P_{\Delta t}\}$ are the resultant vectors of in-plane forces, moment and transverse shear force due to the temperature change (ΔT) in composite matrix whereas $\{N_{\Delta r}\}$, $\{M_{\Delta r}\}$ and $\{P_{\Delta r}\}$ are general in SMA fibre due to the recovery stress.

3.8 Energy Calculation

As s first step, the global displacement vector represented in the matrix form as

$$\{\bar{\delta}\} = \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} = [f]\{\delta\} \tag{3.14}$$

where $[f]$ and $\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y \quad \phi_x \quad \phi_y \quad \psi_x \quad \psi_y\}^T$ are the function of thickness coordinate and the displacement vector at mid plane of the panel, respectively.

The kinetic energy expression (T) of a thermal buckled laminated panel can be expressed as:

$$T = \frac{1}{2}\int_V \rho \left\{\dot{\bar{\delta}}\right\}^T \left\{\dot{\bar{\delta}}\right\} dV \tag{3.15}$$

where , ρ, and $\left\{\dot{\bar{\delta}}\right\}$ are the density and the first order differential of the displacement vector with respect to time, respectively.

Using Eq. (3.14) and (3.15) the kinetic energy can be expressed as

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{\dot{\delta}\right\}^T [f]^T \rho^k [f] \left\{\dot{\delta}\right\} dz \right) dA = \frac{1}{2}\int_A \left\{\dot{\delta}\right\}^T [m] \left\{\dot{\delta}\right\} dA \tag{3.16}$$

where, $[m] = \sum_{k=1}^{N} \int_{z_{k=1}}^{z_k} \left([f]^T \rho^k [f]\right) dz$ is the inertia matrix.

The strain energy (US.E.) of thermally buckled composite panel embedded with SMA fibre can be expressed as:

$$U_{S.E.} = \frac{1}{2}\int_v \{\varepsilon\}_i^T \{\sigma_i\} dV \tag{3.17}$$

By substituting the strain and the stress expression of Eqs. (3.4) and (3.12) into Eq. (3.17) the strain energy expression as:

$$U_{S.E.} = \frac{1}{2}\int_v \{\varepsilon^T\}^k \left(\left[\overline{Q}\right]^k \{\varepsilon\}^k + \{\sigma_{\Delta r}\}^k V_s^k - \left(\left[\overline{Q}\right]_m \{\alpha\}_m V_m\right)^k \Delta T \right) dV \tag{3.18}$$

The work done ($W_{\Delta T}$) by thermal membrane force due to the temperature rise (ΔT) can be expressed in linearized form by following the procedure as adopted by Cook et al. (2000) and conceded to

$$W_{\Delta T} = \frac{1}{2}\int_A \{\varepsilon_G\}^T [D_G]\{\varepsilon_G\} dA \tag{3.19}$$

where $\{\varepsilon_G\}$ is geometric strain and $[D_G]$ is the material property matrix.

3.9 Finite Element Formulation

In this analysis, a nine nodded isoparametric quadrilateral Lagrangian element have 81 degree of freedom (DOFs) per element is employed. The details of the element can be seen in Cook et al. (2000).

The global displacement vector $\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y \quad \phi_x \quad \phi_y \quad \psi_x \quad \psi_y\}^T$ will be represented as stated below by using FEM

$$\{\delta\} = [N_i]\{\delta_i\} \tag{3.20}$$

where $[N_i]$ and $\{\delta_i\}$ are the nodal interpolation function and displacement vector for ith node, respectively.

By substitution of Eq. (3.21) into Eqs. (3.16), (3.18) and (3.19) the kinetic energy, strain energy and the work done expressions can be further expressed as

$$T = \int_A \left([N_i]^T [m][N_i] dA \right) \left\{ \ddot{\delta} \right\} \tag{3.21}$$

$$U_{S.E.} = \frac{1}{2}\int_A \left(\{\delta\}_i^T [B]^T [D][B]\{\delta\}_i\right) dA + \{F_{\Delta r}\} - \{F_{\Delta t}\} \tag{3.22}$$

$$W_{\Delta l} = \frac{1}{2}\int_A \left(\{\delta\}^T [B_G]^T [D_G][B_G]\{\delta\}\right) dA \tag{3.23}$$

where $\{F_{\Delta r}\} = \int_A [B_L]^L \{N_{\Delta r}\} dA$, $\{F_{\Delta l}\} = \int_A [B_L]^L \{N_{\Delta l}\} dA$,

$[D_1] = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} [H]_L^T \left[\overline{Q}\right][H]_L \, dz$

where $[B_L]$ and $[B_G]$ are the product form of the different operator and nodal interpolation function in the linear strain terms and geometric strain terms, respectively.

3.10 Governing Equation

By using the Hamilton's principle, the equation of buckling for the composite will be expressed as:

$$\delta\int_{t_1}^{t_2} L dt = 0 \tag{3.24}$$

where L=T-($U_{S.E.}$ +$W_{\Delta T}$)

3.11 Solution Technique

To analyse the thermal buckling of composite panels embedded with SMA fibres, will be given by

$$(K_s - \lambda_{cr} K_G)\{\delta_s\} = 0 \tag{3.25}$$

For the linear eigenvalue problem without SMA is solved by dropping the term of term of recovery strain.

3.12 Support Condition

The main purpose of boundary condition is to avoid rigid body motion as well as to decrease the number of unknowns of a system for ease in calculating and also the singularity in the matrix equation can be avoided. In the present analysis, kinematical constrain condition are applied as the model is developed using the displacement based finite element i.e. all the unknown are defined of displacement only.

The boundary condition used for the present analysis is expressed below. However the mathematical formulation which is general in nature does not put any limitations.

a) Simple supported boundary conditions (S):

$u_0=w_0=\theta_y= \theta_z =\o_y=\psi_y=0$ at x$=0,a$

$v_0=w_0=\theta_x= \theta_z =\o_x=\psi_x=0$ at $y=0,b$

b) Hinged boundary conditions (H):

$u_0=v_0=w_0=\theta_y= \theta_z =\o_y=\psi_y=0$ at x$=0,a$

$u_0=v_0=w_0=\theta_x= \theta_z =\o_x=\psi_x=0$ at $y=0,b$

c) Clamped boundary condition (C):

$u_0=v_0=w_0=\theta_x =\theta_y= \theta_z =\o_x =\o_y=\psi_x =\psi_y=0$ at x$=0,a$ *and* $y=0,b$

3.13 Computational Investigation

For the computational purpose, a MATLAB code is developed in MATLAB 2010a environment for the thermal buckling analysis of laminated composite panel incorporating SMA fibrei the panel. The code has been developed in such a way that it can easily compute the different type of problem of laminated composite panel embedded with and without SMA fibres.

3.14 Summary

The main aim of this present chapter is to develop a general linear mathematical model for the computer implementation of the proposed problem i.e. the buckling response of thermal buckling laminated composite panel embedded with and without SMA fibre. The need and the necessity of the present problem and their background were discussed in the first section. A few essential assumptions are stated in Section 3.2. Then, in Section 3.3 and 3.4 the geometry of the shell panel and the assumed higher order displacement field were stated. In Section 3.5 the strain-displacement relation is Green-Lagrange sense and subsequent strain vector were evaluated. The mechanics of SMA embedded composite were presented in Section 3.6. The general themo-elastic constitutive relations for laminated composite embedded with SMA fibres and the resultant in-plane thermal forces were discussed in Section 3.7. Then in Section 3.8 various energies and the work done as a result of thermal load were calculated. The linear mathematical model for proposed panel problem was discretised with the help of finite element in Section 3.9. A detail discussion on the solution technique and the necessary assumptions were presented in Section 3.10. Finally, the types of boundary conditions were presented in Section 3.11 for the numerical analysis. Computational investigations were discussed briefly in Section 3.12.

CHAPTER 4

THERMAL BUCKLING ANALYSIS OF LAMINATED COMPOSITE SHELL PANELS

4.1 Introduction

This chapter deals with the study of buckling strength of laminated shell panel by using the proposed linear model under thermal environment. As it is discussed in the previous chapter 1, the laminated shell panel is used in many engineering structures, industries, naval and space projects. Many of structural component of space project like rocket, launch vehicles, etc. are made up of laminated shell panel, that are exposed to harsh environment such as high intensity temperature load due to aerodynamic heating in there service period. Due to such high temperature increase in the structure buckling may be induced in the structure which may leads to malfunctioning of the whole structure. So the mathematical model should be developed in such a fashion so that it is capable enough to carry the effect temperature on structure and include the true geometrical modifications. In this present chapter, effort has been made to predict the thermal loads of the composite shell panel of various geometries. It is also necessary to mention that, to explore the original strength of laminated structures, the mathematical model is developed in the framework of the HSDT by taking the nonlinearity in Green-Lagrange sense to incorporate the true geometrical distortion in geometry.

This chapter describes the governing equation of thermally buckled composite shell panels and the solution steps. The proposed model effectiveness is tested with the results that are available in the literature. New results are computed for various geometries and different parameter.

4.2 Governing Equations and Solution

The system governing equilibrium equations for buckling of laminated composite shell panel is expressed as:

$$\{[K_S]-\lambda_{cr}[K_G]\}\{\delta\}=0 \tag{4.1}$$

where, $[K_S]$ is the stiffness matrix, $[K_G]$ is the global stiffness matrix and $\{\delta\}$ is the displacement vector. The Eq. (4.1) has been solved using direct iterative method.

4.3 Result and Discussion

In this part, the thermal buckling temperature of laminated composite shell panels is obtained by taking nonlinear geometry matrix. As a first step, the validation of present developed code has been performed by comparing the results with those available in the literature. In order to check the efficiency of the present numerical model, a detailed parametric study has been done for the shell panel and the results obtained are presented and discussed. The effects of different parameters like the curvature ratio (R/a), the thickness ratio (a/h), the aspect ratio (a/b), the lay-up scheme and the support condition on the buckling behaviour of composite shell panel are analysed and discussed in detail. For the computational purpose, the following composite material properties have been used throughout in the analysis.

Material Property (M1): E_1=76GPa; E_2=5.5GPa; G_{12}=G_{13}=2.3GPa; G_{23}=1.5GPa;

v_{12}=0.34; α_1= –4×10^{-6} ($^{o}C^{-1}$); α_2=76×10^{-6} ($^{o}C^{-1}$); $\alpha_0=1\times10^{-6}/^{0}C$

Material Property (M2): E_2/E_1 = 0.081; G_{12}/E_1 = G_{13}/E_1 = 0.031; G_{23}/E_1 = 0.0304;

v_{12} = 0.21; α_1= –0.21×10-6 ($^{o}C^{-1}$); α_2=16×10^{-6} ($^{o}C^{-1}$); $\alpha_0=1\times10^{-6}/^{0}C$

Material Property (M3): E_1/E_2=40; G_{12}/E_2 =0.6; G_{13}= G_{12}; G_{23}/E_2=0.5; v_{12}=0.25; α_2/α_1=10

Material Property (M4): E_1=181GPa; E_2= E_3=10.3GPa; G_{12}=G_{13}=7.17GPa;

G_{23}=6.21GPa; v_{12}=0.33; α_1= 0.02×10^{-6} ($^{o}C^{-1}$); α_2=22.5×10^{-6} ($^{o}C^{-1}$); $\alpha_0=1\times10^{-6}/^{0}C$

4.3.1 Convergence and Validation Study

The validation and convergence of the present developed model is carried out by taking different numerical examples. As discussed in earlier chapter, the responses are obtained numerically by the developed APDL code in ANSYS and the responses are compared with those published literature. It is important to mention that the thermal buckling load has been obtained in APDL code (ANSYS) is converging at a (20×20) mesh in throughout the analysis. The non-dimensional forms of the critical buckling temperature loads are obtained by using $\lambda_{cr} = \alpha_0 \times T_{cr} \times 10^3$. It is taken same throughout in the analysis if it is not stated elsewhere.

4.3.1.1 Convergence and validation study for single layer thin plate

Fig. 4.1 shows convergence and validation study of thermal buckling temperature (T_{cr}) with different mesh divisions for single layer (M1) thin square plate (*a*/*h*=40) with all edges clamped under uniform temperature field.

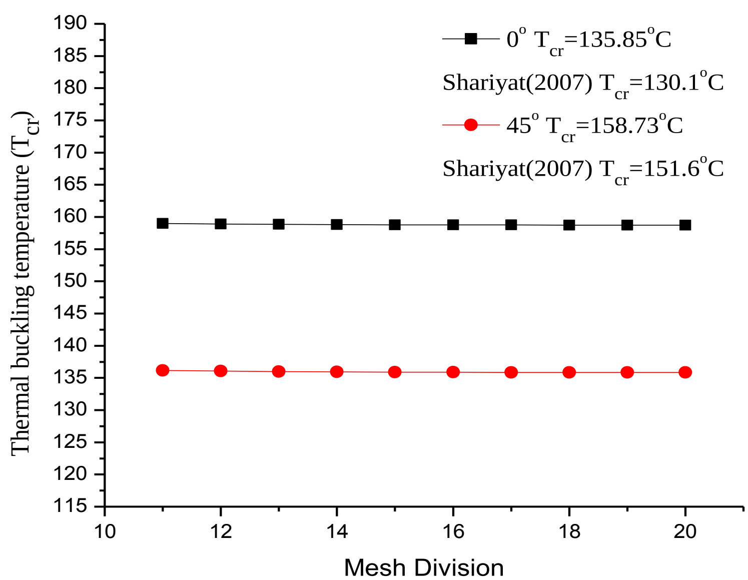

Fig. 4.1 Convergence study of thermal buckling temperature parameter (λ_T) with different mesh divisions

The figure shows that the results are converging well with the mesh refinement and the difference between the present result and the result obtained by Shariyat (2007) is within the expected line.

4.3.1.2 Convergence and validation study for different layup scheme

Fig. 4.2 shows convergence and validation study of thermal buckling temperature load parameter (λ_{cr}) with different mesh division of four-layer symmetric and anti-symmetric laminated (M3) square plate (a/h=100) with lay-up sequence $\left[30^{\circ}/-30^{\circ}\right]_s$, $\left[30^{\circ}/-30^{\circ}\right]_2$, $\left[45^{\circ}/-45^{\circ}\right]_s$ and $\left[45^{\circ}/-45^{\circ}\right]_2$ for all edges hinged (HHHH) subjected to uniform temperature rise.

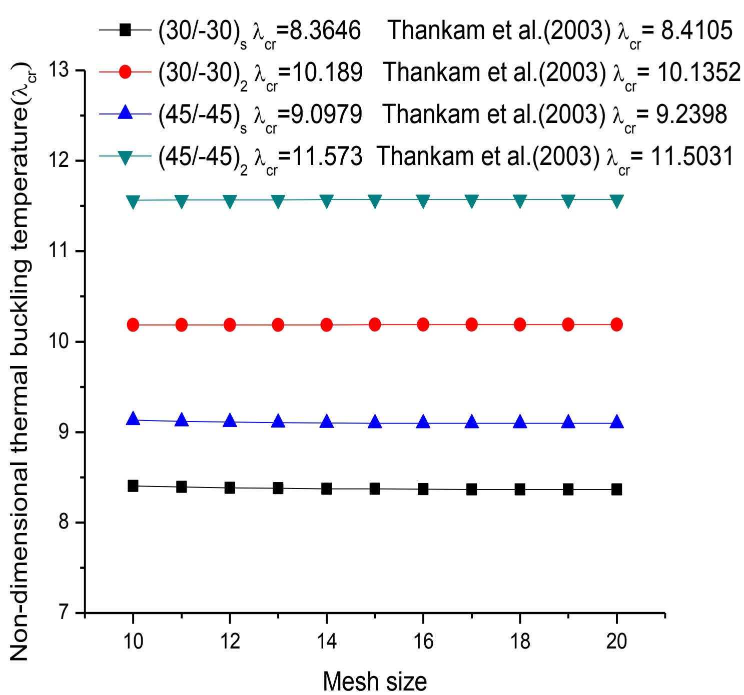

Fig. 4.2 Convergence study of thermal buckling temperature parameter (λ_{cr}) with different mesh divisions and stacking sequence

It is clear from the above figure that the present results are converging well at a (20×20) mesh and the differences are very nominal with the results of Thankam et al. (2003). In addition to that, it is found that anti-symmetric panel has relatively higher buckling load parameter with respect to symmetric angle ply.

4.3.1.3 *Convergence and validation study for different boundary conditions*

Another convergence and validation study is reported in Fig 4.3 which shows thermal buckling load parameter (λ_{cr}) for different mesh division of four-layer anti-symmetric laminated square plate (a/h=100) with lay-up sequence $\left[45^{\circ}/-45^{\circ}\right]_2$ subjected to different boundary conditions (SSSS, HHHH, CCCC) under uniform temperature rise. Material properties used for the computational purpose is M3.

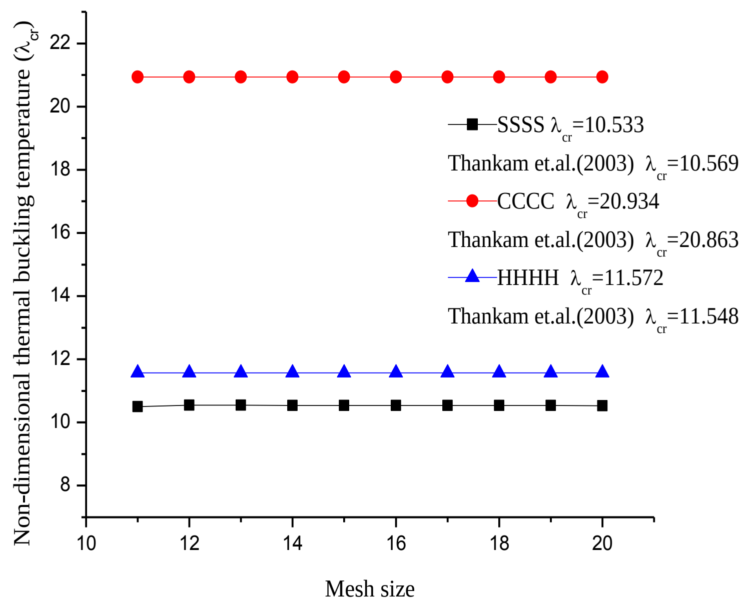

Fig.4.3 Convergence and validation study of thermal buckling temperature parameter (λ_T) with different mesh divisions and boundary condition

The above results show excellent convergence as well as validation with the previous published literature (Thankam et al., 2003). It is observed that clamped plate has maximum bucking temperature with respect to other boundary conditions i.e., as the number of constraints increase, the critical buckling temperature increases.

4.3.1.4 Comparison study for different thickness ratios anti-symmetric angle-ply

Comparison study of the thermal buckling load parameter for a simply supported square thin anti symmetric angle-ply $[\pm 45^{o}]_3$ laminated composite flat panel for different thickness ratio (a/h) having material and geometric properties used as M2 and a/b=1 respectively is tabulated in Table 4.1. Table shows the response obtained from the parametric language ANSYS APDL code and that result are well validated with the previous published paper presented in the literature review.

Table 4.1 Buckling temperature variation with thickness ratio variation

	$(\lambda_{cr} = \alpha_0 \times T_{cr} \times 10^3)$				
a/h	Present (HSDT)	Present (ANSYS)	Shen (2001)	Shariyat (2007)	Palazotto (1992)
50	0.2583	0.486	0.4873	0.482	0.4874
60	0.2151	0.339	-	-	-
70	0.1842	0.24984	-	-	-
80	0.1612	0.1916	0.1919	0.1916	0.1919
90	0.1432	0.1516	-	-	-
100	0.1281	0.1229	0.123	0.1228	0.123

The present thermal buckling load parameter (λ_{cr}) is presented in table 4.1 is compared with HSDT result of Shen (2001), laminated layerwise theory of palazotto (1992). From the table it can be easily examined that as the thickness ratio increases the value of load parameter decreases. So it can be concluded that as the thickness of the panel increases temperature required for the buckling of the plate increases while considering the other parameters same.

4.3.1.5 Validation study of the thermal buckling strength for TD and TID material property

Another validation study of thermal buckling load parameter (λ_{cr}) of simply supported laminated shell panel for different aspect ratio, thickness ratio and stacking sequence is tabulated in Table 4.2. Different material property is improvised on the structure which is temperature dependent and temperature independent. For the computation process, the material property taken is assumed to be linear function of uniform temperature whereas Poisson's ratio is assumed to temperature independent.

$$E_1(T) = E_{10}(1 + E_{11}T), \quad E_2(T) = E_{20}(1 + E_{21}T), \quad G_{12}(T) = G_{120}(1 + G_{121}T),$$
$$G_{13}(T) = G_{130}(1 + G_{131}T), \quad G_{23}(T) = G_{230}(1 + G_{231}T),$$
$$\alpha_1(T) = \alpha_{10}(1 + \alpha_{11}T), \quad \alpha_2 T = \alpha_{20}(1 + \alpha_{21}T)$$

and

$$E_{10} / E_{20} = 40, \quad G_{120} / E_{20} = G_{130} / E_{20} = 0.5, \quad G_{230} / E_{20} = 0.2,$$
$$\nu_{12} = 0.25, \alpha_{10} = 10^{-6}, \quad \alpha_{20} = 10^{-5}, \quad E_{11} = -0.5 \times 10^{-3},$$
$$E_{21} = G_{121} = G_{131} = G_{231} = -0.2 \times 10^{-3}, \quad \alpha_{11} = \alpha_{21} = 0.5 \times 10^{-3}$$

For the analysis if temperature independent property is to be considered for the analysis then the following quantities E_{11}, E_{21}, G_{121}, G_{131}, G_{231}, α_{11} and α_{21} are assumed to be zero.

Table 4.2 Variation of thermal buckling load parameter for different parameter

a/b	b/h	Stacking sequence	$\lambda_{cr} = \alpha_0 \times T_{cr} \times 10^3$			
			Temperature independent		Temperature dependent	
			Present	Shariyat(2007)	Present	Shariyat(2007)
1	30	$(\pm 45_2)_T$	0.975	1.062	0.81	0.747
1	50	$(\pm 45_2)_T$	0.390	0.413	0.333	0.341
1.5	30	$(\pm 45_2)_T$	1.357	0.75	0.593	0.427
1	30	$(\pm 45_5)_T$	1.167	1.232	1.003	0.827
1	30	$(0/90)_s$	0.653	0.667	0.564	0.525

It can be easily noted that without changing the aspect ratio if thickness ratio increases the value of buckling load parameter deceases. From the results it is observed that if the total number of plies increases by keeping the thickness of the plate remains the same, thermal buckling temperature increases.

4.3.1.6 Comparison of critical buckling temperature parameter for different thickness ratio

Comparison study of the thermal buckling temperatures for a four layered clamped square anti symmetric $[45^o/-45^o]_2$ angle ply and $[0^o/90^o]_2$cross ply laminate with different thickness ratio (a/h) having laminated shell panel property and the geometric properties as M4 and a/b=1 respectively. Table shows the response obtained from the APDL code and that result are well validated with the previous published paper presented in the literature review.

Table 4.3 Influence of thickness ratio *(a/h)* on the thermal buckling strength

a/h	$T_{cr}\times\alpha_2\times1000$			
	45/-45/45/-45		0/90/0/90	
	Present	Ref	Present	Ref
10	234.584	173.616	261.584	176.452
20	86.238	90.6	92.27925	95.891
30	42.419	43.745	44.4285	45.565
40	24.842	25.387	25.7422	26.273
50	16.218	16.494	16.70715	17.011
100	4.1706	4.213	4.257	4.320

Table 4.3 shows the thermal buckling temperature load parameter as given by λ_{cr}= $T_{cr}\times\alpha_2\times1000$ embedded with/without SMA. As the above result shows the difference in the result is very low because both the result is obtained by using the FSDT model to obtain the pre-buckling temperature. As from above it can be concluded that the as the thickness ratio increases buckling temperature decreases because as thickness of the plate increases the buckling temperature increases.

4.3.1.7 Influence of fibre orientation on the thermal buckling temperature parameter

The variation of thermal buckling strength of single layer thin square plate for all edge clamped boundary condition exposed to uniform temperature field for different orientation of fibre are tabulated in Table 4.4. The plate material properties and the geometric properties used are M1 and a/b=1 and a/h=40 respectively.

Table 4.4 Influence of fibre orientation on the thermal buckling temperature

Fibre orientation	Critical buckling temperature (T_{cr}) (^oC)
0^o	158.73
15^o	155.69
30^o	147.74
45^o	135.85

It is been observed from the table that the critical buckling temperature is decreasing for the increasing value of angle of plate.

4.3.2 Numerical examples

In this section, different numerical example are solved using the developed general model for the shell panel such as cylindrical and spherical shell panel and the effect of geometric and material properties on the response are presented and discussed in details.

4.3.2.1 Cylindrical shell panel

As discussed earlier, the thermal buckling is important phenomenon for the design of the structures. The proposed geometric model is employed to obtain the thermal buckling temperature of the cylindrical shell panel. In this subsection, a detailed parametric study on the thermal strength of the cylindrical panel is presented and discussed.

4.3.2.1.1 Influence of thickness ratio (a/h)on the thermal buckling strength

Table 4.5 shows the variation of the nondimensional buckling temperature ($\lambda_{cr} = \alpha_o \times \lambda_{cr} \times 10^3$) for a six layered of square symmetric laminated cylindrical shell panel with lay-up sequence $[\pm 45^o]_3$ for different thickness ratio (a/h) with simply supported edge boundary condition. For the computational purpose, the composite properties and the geometric properties used are M2 and R_1= R, $R_2=\infty$ and two curvature ratio has taken to study the influence of thickness ratio with the variation of curvature ratio, R/a=1000, 100.

Table 4.5 Influence of thickness ratio *(a/h)* on the thermal buckling strength

a/h	($\lambda_{cr} = \alpha_o \times T_{cr} \times 10^3$)	
	R/a=1000	R/a=100
5	34.12	12.9
8	23.18	11.86
10	17.78	10.06
15	9.897	6.616
20	6.137	4.496
30	2.960	2.366
40	1.721	1.429
50	1.120	0.942
80	0.447	0.3895
100	0.288	0.2536

From the above Table 4.5 it can be observed that as thickness ratio increases the thermal buckling temperature increases. As the curvature ratio increases buckling temperature increases for higher cylindrical curved panel so it can be concluded that as the curvature turns towards flatness the buckling temperature required is higher.

4.3.2.1.2 Influence of curvature ratio (R/a) on the thermal buckling strength

The nondimensional buckling temperatures ($\lambda_{cr} = \alpha_o \times T_{cr} \times 10^3$) of a simply supported square symmetric angle ply $[\pm 45^o]_3$ and cross ply $[0/90]_3$ laminated cylindrical shell panel for different curvature ratio (R/a) are tabulated in Table 4.5. For the computational purpose, the composite properties and the geometric properties used are M2 and R_1= R, R_2=∞.

The value of the nondimensional buckling temperature parameter (λ_{cr}) show higher for the cross ply $[0/90]_3$ and lower values for angle ply $[\pm 45^o]_3$. Buckling temperature decreases for increased value of curvature ratio (R/a). For the same length of cylindrical shell panel as the curvature increases buckling temperature increases so, as curvature moves towards the flatness, temperature required to bend the panel is high. So, more flatter the curve higher amount of temperature is required for the buckling of the panel.

Table 4.6 Influence of curvature ratio *(R/a)* on the thermal buckling strength

Curvature Ratio	Buckling Temperature (λ_T)	
R/a	$[\pm 45^o]_3$	$[0/90]_3$
10	0.89926	2.702
50	0.30598	1.8765
100	0.29223	1.8538
150	0.28971	1.8489
200	0.28884	1.8473
300	0.28821	1.8462
400	0.28799	1.8458
500	0.28789	1.8456
800	0.28778	1.8453
1000	0.28776	1.8451

4.3.2.1.3 *Influence of material property on the thermal buckling strength*

The change of thermal buckling temperature parameter of laminated cylindrical shell panel for different aspect ratio, thickness ratio and stacking sequence is tabulated in Table 4.3 with simply supported boundary condition. Different material property is improvised on the structure which is temperature dependent and temperature independent. For the computation process, the material property taken is assumed to be linear function of uniform temperature whereas poisson's ratio is assumed to temperature independent.

$$E_1(T) = E_{10}(1+E_{11}T), \quad E_2(T) = E_{20}(1+E_{21}T), \quad G_{12}(T) = G_{120}(1+G_{121}T),$$
$$G_{13}(T) = G_{130}(1+G_{131}T), \quad G_{23}(T) = G_{230}(1+G_{231}T),$$
$$\alpha_1(T) = \alpha_{10}(1+\alpha_{11}T), \quad \alpha_2 T = \alpha_{20}(1+\alpha_{21}T)$$

and

$$E_{10}/E_{20} = 40, \quad G_{120}/E_{20} = G_{130}/E_{20} = 0.5, \quad G_{230}/E_{20} = 0.2,$$
$$\nu_{12} = 0.25, \alpha_{10} = 10^{-6}, \quad \alpha_{20} = 10^{-5}, \quad E_{11} = -0.5\times10^{-3},$$
$$E_{21} = G_{121} = G_{131} = G_{231} = -0.2\times10^{-3}, \quad \alpha_{11} = \alpha_{21} = 0.5\times10^{-3}$$

For the analysis if temperature independent property is to be taken then the following terms is taken zero values E_{11}, E_{21}, G_{121}, G_{131}, G_{231}, α_{11} and α_{21}.

Table 4.7 Variation of thermal buckling temperature for different parameter

a/b	b/h	Stacking sequence	Temperature independent	Temperature dependent
			Present	Present
1	30	$(\pm 45_2)_T$	2.087	1.8214
1	50	$(\pm 45_2)_T$	0.885	0.76305
1.5	30	$(\pm 45_2)_T$	4.523	3.7019
1	30	$(\pm 45_5)_T$	7.1556	6.4617
1	30	$(0/90)_s$	16.621	14.961

Result is calculated for different aspect ratio, thickness ratio and different stacking sequence. From the above table it can be easily noted that without changing the aspect ratio if thickness ratio increases the value of buckling parameter goes on deceasing. The influence of temperature parameter in the material property is for temperature independent material buckling temperature parameter is higher with respect to temperature dependent material property.

4.3.2.1.4 Convergence study of thermal buckling temperature parameter

Fig 4.4 shows convergence and validation study of thermal buckling temperature load parameter with different mesh division of four-layer anti-symmetric laminated cylindrical shell panel with lay-up sequence $\left[45^\circ / -45^\circ\right]_2$ for a different boundary condition (SSSS, CCCC) subjected to uniform temperature rise a using simulation model developed in ANSYS APDL environment. The material and geometry properties used are M3 and the panel dimensions is a/b=1, a/h=100 respectively and R_1= R, $R_2=\infty$.

From the Fig.4.4 thermal buckling temperature is high for clamped boundary condition with respect to simply supported boundary condition. Here it can be concluded that as the number of constrains increases the buckling temperature increases.

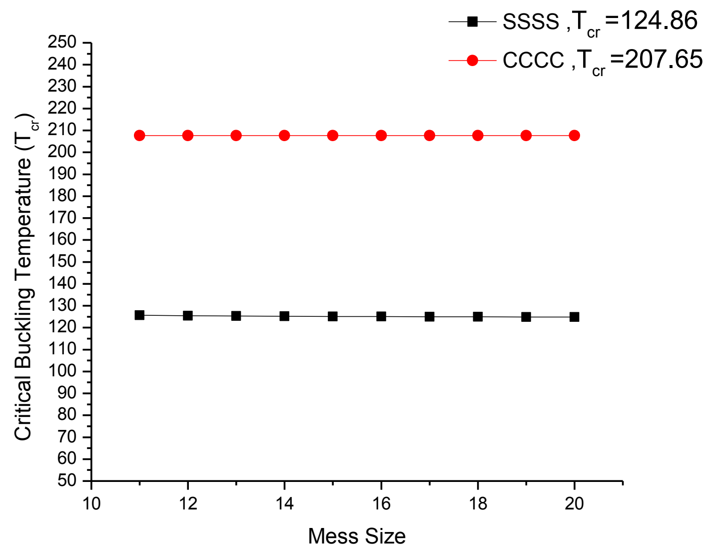

Fig. 4.4 Convergence study of thermal buckling temperature parameter (λ_T) with different mesh divisions and boundary condition

4.3.2.2 Spherical shell panel

As discussed earlier, the thermal buckling is important phenomenon for the design of the structures. The proposed geometric model is employed to obtain the thermal buckling temperature of the spherical shell panel. In this subsection, a detailed parametric study on the thermal strength of the spherical panel is presented and discussed.

4.3.2.2.1 *Influence of thickness ratio (a/h)on the thermal buckling strength*

The nondimensional buckling temperature ($\lambda_T = \alpha_o\ \lambda_{cr}\ 10^3$) of a simply supported square symmetric angle ply $[\pm 45^o]_3$ laminated spherical shell panel for different thickness ratio (a/h) are tabulated in Table 4.5. For the computational purpose, the composite properties and the geometric properties used are M2 and R_1=R_2= R and R/a=1000, 100.

Table 4.8 Influence of thickness ratio *(a/h)* on the thermal buckling strength

a/h	$(\lambda_T = \alpha_o \times \lambda_{cr} \times 10^3)$	
	R/a=1000	R/a=100
5	34.12	12.883
8	23.175	11.875
10	17.779	10.074
15	9.894	6.626
20	6.135	4.506
30	2.9598	2.3772
40	1.7206	1.4406
50	1.120	0.9610
80	0.4469	0.4018
100	0.2878	0.2660

From the above Table 4.8 it can be observed that as thickness ratio increases the thermal buckling temperature increases. And as the curvature ratio increases buckling temperature increases.

4.3.2.2.2 *Influence of curvature ratio (R/a)on the thermal buckling strength*

The nondimensional buckling temperature ($\lambda_T = \alpha_o \lambda_{cr} 10^3$) of a square symmetric angle ply $[\pm45^o]_3$ and cross ply $[0/90]_3$ laminated spherical shell panel for different thickness ratio (a/h) are tabulated in Table 4.9 with simply supported boundary condition. For the computational purpose, the composite properties and the geometric properties used are M2 and $R_1 = R_2 = R$.

Table 4.9 Influence of curvature ratio *(R/a)* on the thermal buckling strength

Curvature Ratio	Buckling Temperature (λ_T)	
R/a	$[\pm45^o]_3$	$[0/90]_3$
10	3.1633	6.5545
50	0.3643	1.9672
100	0.3059	1.8745
150	0.2957	1.8576
200	0.2921	1.8517
300	0.2896	1.8475
400	0.2887	1.8461
500	0.2883	1.8454
800	0.2878	1.8447
1000	0.2877	1.8445

Table 4.9 shows the variation of result with respect to curvature ratio. The value of the nondimensional buckling temperature parameter (λ_T) show high temperature values for the cross ply $[0/90]_3$ with respect to angle ply $[\pm 45^o]_3$. As from table it can be said that buckling temperature decreases as the increases value of curvature ratio (R/a).

4.3.2.2.3 *Influence of material property on the thermal buckling strength*

The change in thermal buckling temperature parameter of laminated shell panel for different aspect ratio, thickness ratio and stacking sequence is tabulated in Table 4.3 with simply supported boundary condition. Different material property is improvised on the structure which is temperature dependent and temperature independent. For the computation process, the material property taken is assumed to be linear function of uniform temperature whereas poisson's ratio is assumed to temperature independent.

$$
\begin{aligned}
&E_1(T) = E_{10}(1+E_{11}T), \quad E_2(T) = E_{20}(1+E_{21}T), \quad G_{12}(T) = G_{120}(1+G_{121}T),\\
&G_{13}(T) = G_{130}(1+G_{131}T), \quad G_{23}(T) = G_{230}(1+G_{231}T),\\
&\alpha_1(T) = \alpha_{10}(1+\alpha_{11}T), \quad \alpha_2 T = \alpha_{20}(1+\alpha_{21}T)
\end{aligned}
$$

and

$$
\begin{aligned}
&E_{10}/E_{20} = 40, \quad G_{120}/E_{20} = G_{130}/E_{20} = 0.5, \quad G_{230}/E_{20} = 0.2,\\
&\nu_{12} = 0.25, \alpha_{10} = 10^{-6}, \quad \alpha_{20} = 10^{-5}, \quad E_{11} = -0.5\times 10^{-3},\\
&E_{21} = G_{121} = G_{131} = G_{231} = -0.2\times 10^{-3}, \quad \alpha_{11} = \alpha_{21} = 0.5\times 10^{-3}
\end{aligned}
$$

For the analysis if temperature independent property is to be taken the following terms should be zero E_{11}, E_{21}, G_{121}, G_{131}, G_{231}, α_{11} and α_{21}.

Result is calculated for different aspect ratio, thickness ratio and different stacking sequence. From the above table it can be easily noted that without changing the aspect ratio if thickness ratio increases the value of buckling parameter goes on deceasing. The influence of temperature parameter in the material property is for temperature independent material buckling temperature parameter is higher with respect to temperature dependent material property.

Table 4.10 Variation of thermal buckling temperature for different parameter

a/b	b/h	Stacking sequence	Temperature independent	Temperature dependent
			Present	present
1	30	$(\pm 45_2)_T$	2.087	1.82
1	50	$(\pm 45_2)_T$	0.885	0.762
1.5	30	$(\pm 45_2)_T$	0.884	1.82
1	30	$(\pm 45_5)_T$	2.3043	2.0026
1	30	$(0/90)_s$	16.628	14.966

4.3.2.2.4 *Convergence study of thermal buckling temperature parameter*

Fig 4.3 shows convergence and validation study of thermal buckling temperature load parameter with different mesh division of four-layer anti-symmetric laminated cylindrical shell panel with lay-up sequence $\left[45^{\circ}/-45^{\circ}\right]_2$ for a different boundary condition (SSSS, CCCC) subjected to uniform temperature rise a using simulation model developed in ANSYS APDL environment. The material and geometry properties used are M3 and the panel dimensions is a/h=100 respectively and $R_1= R_2=R$.

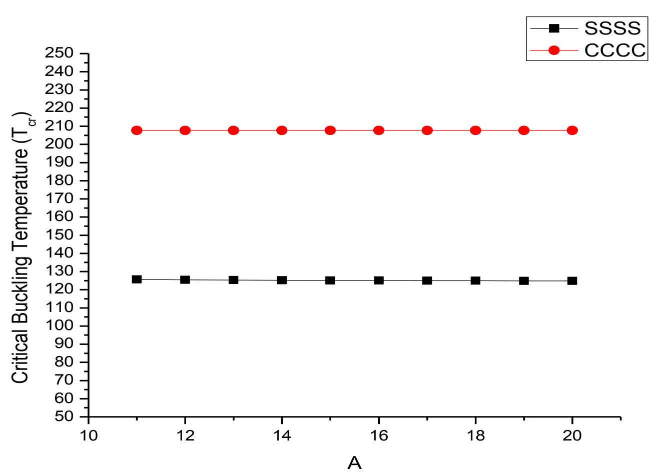

Fig. 4.5 Convergence study of thermal buckling temperature parameter (λ_T) with different mesh divisions and boundary condition

Here it can be concluded that as the number of constrains increases the buckling temperature increases.

4.4 Conclusions

Based on the above numerical analysis the following conclusions are made:

a) The parametric study indicates that the buckling temperature greatly dependent on the composite properties, the support condition and the type of laminated scheme (symmetric or antisymmetric and cross-ply and angle ply).

b) The buckling temperatures are higher for the symmetrical cross-ply and angle-ply laminate in comparison to anti-symmetric laminates.

c) The buckling strength of shell panels increases with decreases in the curvature ratio, thickness ratio.

d) Material property has a great effect on the buckling strength of material.

CHAPTER 5

THERMAL BUCKLING ANALYSIS OF LAMINATED SHELL PANELS EMBEDDED WITH SMA FIBRES

5.1 Introduction

In this chapter, the buckling strength of laminated composite shell embedded with SMA fibres exposed to thermal environment have been studied using the proposed nonlinear model. The geometrical nonlinearity in Green-Lagrangian sense and material nonlinearity in the SMA are incorporated in the model through the strain displacement relations and the constitutive relations are discussed in Chapter 3. It is well known from the earlier discussion that, the excess thermal deformation and the induced thermal stress change the structural geometry, which considerably affects the performance of the laminated structures. Since the laminated structure are very flexible in nature, the induced thermal deformations and stress play an important role in their practical design. In order to suppress these deformations and increases the life of the laminated structures against thermal loading passive treatments were used earlier, but they suffer from weight penalty. In order to increases these performance smart materials are suitable alternative. In recent years, many studies have been carried out by different researchers to improve the structural performances against thermal loading using SMA fibres as a smart material. In this analysis as effort has been made to predict the enhancement of the thermal buckling strength of the plate embedded with SMA fibres considering the excess thermal deformation of the shell panel by taking the geometric matrix and geometric nonlinearity in Green-Lagrange sense based on the HSDT.

The purpose of this chapter is to derive the governing system equation of laminated composite plate embedded with SMA fibres by taking the geometric nonlinearity in Green-Lagrange sense and the material nonlinearity in SMA through a marching technique. The solution techniques are outlined for the first time derived governing system equations. A number of numerical examples have been solved for plate and discussed. The efficiency and necessity of the present nonlinear model has been shown by comparing the result with those available in the literature.

5.2 Governing System Equation and solution

The governing equations of motion for thermal buckling composite plate embedded with SMA fibres are obtained using Eq. (3.12). These are rewritten elaborately as follows:

$$\left([K]_L + [K]_{\Delta r} - \lambda_{cr}[K_G]\right)\{\delta\} = 0 \tag{5.1}$$

where, $\{\delta\}$ is the global displacement vector, $[K]_L$, $[K]_{\Delta r}$ and $[K_G]$ are the global linear stiffness matrix, geometric stiffness matrix of SMA due to recovery stress and global geometric matrix due to the uniform temperature rise, respectively.

The matrix equation (5.1) is solved using direct iterative method following the same steps as discussed in Chapter 4.

5.3 Result and Discussion

A computer code has been developed in MATLAB R2010a for laminated composite plate embedded with SMA fibres by taking the geometric nonlinearity in Green-Lagrange sense and the material nonlinearity in SMA due to the temperature change. In the present study a uniform temperature variation has been taken for the composite panel through thickness. The composite matrix properties are taken as a function of temperature. The SMA properties are evaluated by using marching technique with small increment in temperature whereas reference temperature assumed to be 24^{o}C. A validation and convergence test has been done to show the efficiency of the present nonlinear model. The effect of different parameters on the buckling temperature have been obtained in detail and discussed. The stacking sequence, geometrical and the material properties are taken same as the Park et al. (2004) for the comparison and convergence purpose. The mechanical and thermal properties matrix and SMA fibres are presented in Table 5.1 and are used for the analysis unless started otherwise.

Table 5.1 Composite/ SMA material properties

Graphite/epoxy	(Park *et al.* 2004)	SMA fibre	(Park *et al.* 2004)
E_1 (GPa)	155	E_s	Park *et al.* 2004
E_2 (GPa)	8.07	σ_r	Park *et al.* 2004
G_{12} (GPa)	4.55		
G_{23} (GPa)	3.25		
ν_{12}	0.22	ν_s	0.33
α_1 (1/^{o}C)	-0.07×10^{-6}	α(1/^{o}C)	10.26×10^{-6}
α_2 (1/^{o}C)	30.1×10^{-6}		

5.3.1 Convergence and validation study

In this section the convergence and the validation study for the buckling of laminated composite shell panel embedded with SMA fibres are obtained using the proposed nonlinear model. As pointed out in preceding chapter 4, the present results are compared with the published results obtained using numerical techniques. Based on the convergence study a (5×5) mesh is used to compute the result.

5.3.1.1 Convergence study of buckling temperature of composite panel embedded with SMA fibres

A convergence test of buckling temperature of a simply supported rectangular laminated composite flat panel embedded with SMA fibres with different mesh divisions is presented in Fig. 5.1. The buckling temperature has been computed for 10% volume fraction and 2% prestrain value of SMA embedded lamina. It is observed that the present result well converge as the mesh size increases. A (5×5) mesh is used for the comparisons and validation of result.

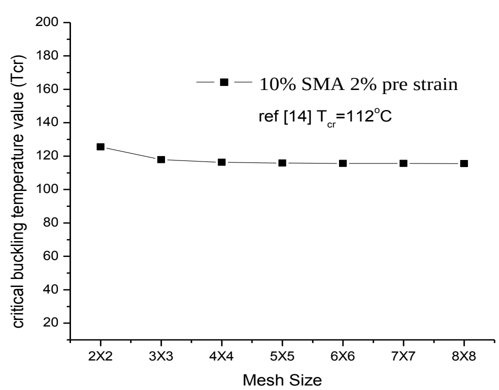

Fig. 5.1 Convergence study of critical buckling temperature (T_{cr})

5.3.1.2 Influence of thickness ratio (a/h) on the buckling temperature parameter

Influence of thickness ratio (a/h) on the nondimensional buckling temperature of a square symmetric angle-ply $(0/\pm45/90)_S$ and $(0/\pm90/0)_S$ laminated composite plate embedded with SMA fibre is depicted in table 6.1 subjected simply supported boundary condition. The SMA is 2% prestrain. It is can be concluded that the buckling temperature decreases as thickness ratio increases.

Table 5.1 Influence of thickness ratio on buckling temperature load parameter

a/*h*	$(0/45/-45/90)_s$		$(0/90/90/0)_s$	
	SSSS	CCCC	SSSS	CCCC
10	141.3509	192.5468	7.7964	11.1488
20	96.4102	180.3748	4.8130	11.2242
40	53.053	120.0852	2.5690	7.3339
50	43.0139	100.9401	2.0755	6.1458
80	27.3406	67.7467	1.144	4.1113
100	21.9784	55.4783	1.0556	3.3607

5.4Conclusion

Thermal buckling strength of curved laminated composite shell panel embedded with SMA fibres has been studied using the proposed linear model. The geometrical nonlinearity is modelled in Green-Lagrange sense incorporating all the higher order terms arising in the mathematical formulation in the framework of HSDT. The material nonlinearity is introduced for the SMA fibres due to the temperature increment through a marching technique. The composite material properties are assumed to be temperature dependent along the SMA. Influences of various geometrical parameters, the support condition have been studied.

Based on the numerical results following conclusion drawn:

a) The buckling temperature decreases with the increases in thickness ratio.
b) For the clamped boundary condition, buckling load parameter retains higher values.

CHAPTER 6

CLOSURES

6.1 Concluding Remarks

In this present work, the thermal buckling behaviour of composite shell panel embedded with and without SMA fibers are examined. A general mathematical model based on the HSDT mid-plane kinematics is considered to obtain the true responses of composite shell panels. The geometric instability is considered through the nonlinear Green-Lagrange strain displacement relations to account the excess thermal deformation occurs during the buckling. A suitable finite element model is proposed and implemented to solve the present developed model. A detailed parametric study has been carried out by solving various numerical examples of laminated composite shell panel of different geometries. The more specific conclusions as a result of the present investigation are stated below:

- In the present model, all the higher order terms are retained in the mathematical formulation for more accurate prediction of the structural behaviour. It is important to mention that, the Green-Lagrange type strain displacement relation is considered to take into account the geometrical nonlinearity arising in the curved panel due to excess deformation for evaluation of geometric stiffness matrix in buckling analysis. It is concluded that the geometric nonlinearity modelled in von-Karman sense based on the FSDT and/or the HSDT is unrealistic in nature for the structures having severe geometric alteration.
- A linear finite element is proposed and implemented for the discretisation of the shell panel model by using a nine noded isoparametric Lagrangian element having nine degrees of freedom per node. The governing system equations of buckling are derived and solved to obtain the desired responses.
- Convergence study is performed by refining the mesh density. The comparison study for different cases indicates the necessity and requirement of the present mathematical model for an accurate prediction of the structural behaviour.
- The thermal buckling strength of laminated composite cylindrical, spherical and flat panel has been examined by taking the uniform temperature field throughout the

thickness. The non-dimensional critical buckling load parameters are obtained by solving the linear eigenvalue problem. Effects of the thickness ratio, the curvature ratio, the lay-up scheme and the support condition are studied in details.

- The non-dimensional critical buckling temperature parameter of angle-ply is higher than the cross-ply laminations.
- The non-dimensional buckling load/parameter increases with the increase in number of layers and the thickness ratios but it is important to mention that in some of the cases the responses are following a reverse trend for the small strain and large deformations. The responses are decreasing with the increase in curvature ratio and mixed type of behaviour is observed for support conditions and lay-up schemes.
- The SMA embedded composite shell panels are having higher buckling temperature in comparison to the composite shell panel without SMA fibers. Effect of the thickness ratio is studied in details along with the effect of stacking sequence.

6.2 Significant Contribution of the Thesis

The contributions of the present research work are as follows:

- A general mathematical model of curved panel has been developed in the framework of the HSDT mid-plane kinematics by taking all the higher order terms in the mathematical formulation for more accurate prediction of thermal buckling behaviour of laminated composite panels.
- A finite element method is proposed and implemented for the discretisation of the shell panel model by using a nine noded isoparametric Lagrangian element with nine degrees of freedom per node.
- In buckling case, Green-Lagrange type strain displacement relation is considered to account the geometrical nonlinearity arising in the curved panel due to excess thermal deformation for evaluation of geometry stiffness matrix.
- A computer code is developed in MATLAB environment based on the prosed mathematical formulation and finite element steps.
- Further, the panel model has been developed in the commercial FE software ANSYS by using APDL code. An eight noded isoparametric serendipity shell element (SHELL281) is employed to discretise the simulation model.

- Different numerical examples have been considered to show the efficacy of the developed mathematical and simulation model. The convergence and comparison study for buckling behaviour of laminated panel is presented.
- The effects of various panel geometries (spherical, cylindrical and flat) and other geometrical parameters (thickness ratios, curvature ratios, aspect ratios, lamination schemes and support conditions) on buckling responses are studied.
- Thermal Buckling strength of the laminated composite spherical, cylindrical and flat panel is obtained by taking the uniform temperature throughout the thickness. The temperature dependent and independent composite material properties with and without SMA are taken for the present analysis.

Finally, it is understood from the previous discussions that the developed general mathematical panel model in the framework of the HSDT would be useful for more accurate analysis of laminated composite structures exposed to excess thermal deformations. It is important to mention that the geometry matrix associated in the buckling has been taken in Green-Lagrange sense for the analysis. On the other hand, it is observed that the present developed FE model in ANSYS environment is also capable to solve any buckling problem easily and with less computational time. And hence, the present analysis would be useful for practical design of the structure.

6.3 Future Scope of the Research

- The present study has been done by using the linear mathematical model which can be extended for nonlinear analysis of laminated composite.
- The present study can be extended to investigate the nonlinear thermo-mechanical post-buckling behavior of laminated composite structures by taking temperature dependent material properties based on nonlinear mathematical model.
- An experimental study on buckling of laminated composite panels will give better understanding about the present developed numerical model.

- By extending the present model, a nonlinear mathematical model can be developed to study the behaviour of laminated composite and sandwich structures in thermal and/or hygro-thermal environment.
- It will be interesting to study the flutter characteristics considering the aerodynamic and acoustic loading that arises frequently in the practical cases.

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